gl_n^ϵ

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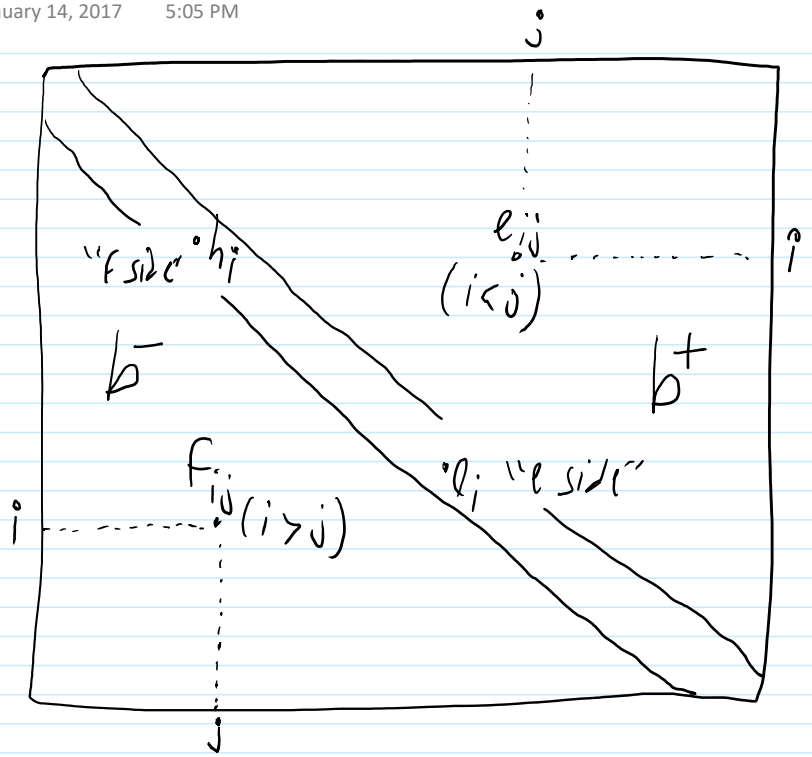

$$\langle e_{ij}, f_{kl} \rangle = \delta_{jk}\delta_{il}$$

$$\langle h_i, h_j \rangle = \delta_{ij}$$

$$[e_{ij}, e_{kl}] = \delta_{jk} e_{il} - \delta_{li} e_{kj} \qquad [f_{ij}, f_{kl}] = \epsilon\delta_{jk} f_{il} - \epsilon\delta_{li} f_{kj}$$

$$[h_i, e_{jk}] = (\delta_{ij} - \delta_{ik}) e_{jk} \qquad [h_i, f_{jk}] = \epsilon(\delta_{ij} - \delta_{ik}) f_{jk}$$

$$[h_i, h_j] = 0 \qquad [h_i, h_j] = 0$$

$$[h_i, h_j] = 0 \qquad [h_i, f_{jk}] = -(\delta_{ij} - \delta_{ik}) f_{jk}$$

$$[e_{ij}, h_k] = \epsilon(\delta_{ik} - \delta_{jk}) e_{ij}$$

$$[e_{ij}, f_{kl}] = \delta_{jk}\left([i<l]\epsilon e_{il} + [i=l](h_i - \epsilon l_i) + [i>l] f_{il}\right) - \delta_{li}\left([k<j]\epsilon e_{kj} + [k=j](h_j - \epsilon l_j) + [k>j] f_{kj}\right)$$

2nd pass: $a \wedge b := a \otimes b - b \otimes a$

$$[e_{ij}, e_{kl}] = \delta_{jk} e_{il} - \delta_{li} e_{kj} \qquad f e_{ik} \overset{?}{=} \sum_{i \le j \le k} e_{ij} \wedge e_{jk}$$

From glnenb

```
P[e_{i_,j_}, f_{k_,l_}] := χ[j == k ∧ i == l]; P[f_{k_,l_}, e_{i_,j_}] := χ[j == k ∧ i == l];
P[e__, h_] = 0; P[h_, e__] = 0;
P[g_, f__] = 0; P[f__, g_] = 0;
P[g_i_, h_j_] := χ[i == j]; P[h_j_, g_i_] := χ[i == j];
P[(e | g)__, (e | g)__] = 0;
P[(f | h)__, (f | h)__] = 0;
```

```
{s1 → -1/2, s2 → -1/2, s3 → 1, s4 → 1, s5 → -1/2, s6 → -1/2, s7 → 1, s8 → 1, s9 → 1, s10 → 1, s11 → 1} /. Rule → Set;
B[e_{i_,j_}, e_{k_,l_}] := χ[j == k] e_{i,l} - χ[l == i] e_{k,j};
B[g_i_, e_{j_,k_}] := (χ[i == j] - χ[i == k]) e_{j,k}; B[e_{j_,k_}, g_i_] := -(χ[i == j] - χ[i == k]) e_{j,k};
B[g_, g_] = 0;

B[f_{i_,j_}, f_{k_,l_}] := ϵ χ[j == k] f_{i,l} - ϵ χ[l == i] f_{k,j};
B[h_i_, f_{j_,k_}] := -ϵ s1 (χ[i == j] - χ[i == k]) f_{j,k}; B[f_{j_,k_}, h_i_] := ϵ s1 (χ[i == j] - χ[i == k]) f_{j,k};
B[h_, h_] = 0;

B[g_, h_] = 0; B[h_, g_] = 0;
B[e_{i_,j_}, h_k_] := s2 ϵ (χ[i == k] - χ[j == k]) e_{i,j}; B[h_k_, e_{i_,j_}] := -ϵ s2 (χ[i == k] - χ[j == k]) e_{i,j};
B[g_i_, f_{j_,k_}] := s11 (χ[i == j] - χ[i == k]) f_{j,k}; B[f_{j_,k_}, g_i_] := -s11 (χ[i == j] - χ[i == k]) f_{j,k};
B[e_{i_,j_}, f_{k_,l_}] :=
  Expand[s3 χ[j == k] (χ[i < l] ϵ e_{i,l} + χ[i == l] (s7 h_i - s5 ϵ g_i) + s9 χ[i > l] f_{i,l}) -
    s4 s3 χ[l == i] (χ[k < j] ϵ e_{k,j} + χ[k == j] (s8 h_j - s6 ϵ g_j) + s10 χ[k > j] f_{k,j})];
B[f_{i_,j_}, e_{k_,l_}] := Expand[-B[e_{k,l}, f_{i,j}]]
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Does it make sense to set $g_i := e_{ii}$, $h_j := -\frac{1}{2} f_{jj}$?

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P[e_{i_,j_}, f_{k_,l_}] := χ[j == k ∧ i == l]; P[f_{k_,l_}, e_{i_,j_}] := χ[j == k ∧ i == l];
P[e__, h_] = 0; P[h_, e__] = 0;
P[g_, f__] = 0; P[f__, g_] = 0;
P[g_i_, h_j_] := χ[i == j]; P[h_j_, g_i_] := χ[i == j];
P[(e | g)__, (e | g)__] = 0;
P[(f | h)__, (f | h)__] = 0;
```

```
{s1 → -1/2, s2 → -1/2, s3 → 1, s4 → 1, s5 → -1/2, s6 → -1/2, s7 → 1, s8 → 1, s9 → 1, s10 → 1, s11 → 1} /. Rule → Set;
B[e_{i_,j_}, e_{k_,l_}] := χ[j == k] e_{i,l} - χ[l == i] e_{k,j};
B[g_i_, e_{j_,k_}] := (χ[i == j] - χ[i == k]) e_{j,k}; B[e_{j_,k_}, g_i_] := -(χ[i == j] - χ[i == k]) e_{j,k};
B[g_, g_] = 0;

B[f_{i_,j_}, f_{k_,l_}] := ϵ χ[j == k] f_{i,l} - ϵ χ[l == i] f_{k,j};
B[h_i_, f_{j_,k_}] := -ϵ s1 (χ[i == j] - χ[i == k]) f_{j,k}; B[f_{j_,k_}, h_i_] := ϵ s1 (χ[i == j] - χ[i == k]) f_{j,k};
B[h_, h_] = 0;

B[g_, h_] = 0; B[h_, g_] = 0;
B[e_{i_,j_}, h_k_] := s2 ϵ (χ[i == k] - χ[j == k]) e_{i,j}; B[h_k_, e_{i_,j_}] := -ϵ s2 (χ[i == k] - χ[j == k]) e_{i,j};
B[g_i_, f_{j_,k_}] := s11 (χ[i == j] - χ[i == k]) f_{j,k}; B[f_{j_,k_}, g_i_] := -s11 (χ[i == j] - χ[i == k]) f_{j,k};
B[e_{i_,j_}, f_{k_,l_}] :=
  Expand[s3 χ[j == k] (χ[i < l] ϵ e_{i,l} + χ[i == l] (s7 h_i - s5 ϵ g_i) + s9 χ[i > l] f_{i,l}) -
    s4 s3 χ[l == i] (χ[k < j] ϵ e_{k,j} + χ[k == j] (s8 h_j - s6 ϵ g_j) + s10 χ[k > j] f_{k,j})];
B[f_{i_,j_}, e_{k_,l_}] := Expand[-B[e_{k,l}, f_{i,j}]]
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